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EMERGENT PHASES IN A DISCRETE FLOCKING MODEL WITH RECIPROCAL INTERACTION

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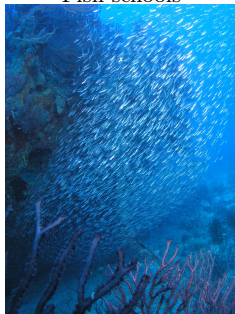
MONDAY, MARCH 17th

DPG Meeting - Regensburg 2025 (DY 3.4)

- 1 Introduction
- 2 The two-species active Ising model
- 3 The two-species active Ising model with species flip
- 4 Conclusion

Introduction: collective motion to Vicsek model

Fish schools



Becco et al., Physica A (2006)

Bird flocks

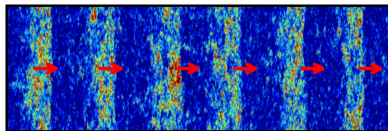


Ballerini et al., PNAS (2008)

- Collective motion: out-of-equilibrium phenomenon with internal energy consumption to self-propel.
- Flocking transition: spontaneous emergence of synchronized motion of large clusters (large densities, low noise).
- First theoretical model: the Vicsek model (1995), where particles follow the ferromagnetic alignment:

$$\begin{aligned}\theta_i(t+1) &= \langle \theta(t) \rangle_r + \eta \xi_i(t) \\ \mathbf{x}_i(t+1) &= \mathbf{x}_i(t) + v \mathbf{e}_i(t+1)\end{aligned}$$

with $\mathbf{e}_i(t+1)$ in the direction $\theta_i(t+1)$, $\xi_i(t)$ a white noise.



- Spontaneous breaking of the continuous symmetry, via a liquid-gas phase transition.

T. Vicsek et al., PRL **75**, 1226 (1995).

A. P. Solon et al., PRL **114**, 068101 (2015).

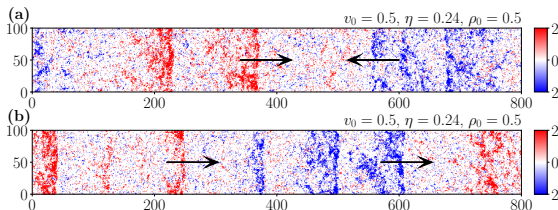
Introduction: the two-species Vicsek model and discrete versions

- Introduction of two species defined by a spin $s_i = \pm 1$, modifying the alignment rule:

$$\langle \mathbf{e}_i \rangle_r = \sum_j J_{ij} \mathbf{e}_j$$

with $J_{ij} = s_i s_j$ (ferromagnetic if same species, anti-ferromagnetic if different species).

- Main results of the two-species Vicsek model (TSVM):
 - Two main states showing antiparallel (APF) and parallel (PF) flocking.



- Stochastic switching between both states, liquid phase only in APF state.

S. Chatterjee, M. Mangeat, C.-U. Woo, H. Rieger, and J. D. Noh, PRE **107**, 024607 (2023).

- Two discretized models, discussed next:
 - Replace the continuous angle θ_i by a two-state spin $\sigma_i = \pm 1$.
 - Add a second ferromagnetic interaction between species $s_i = \pm 1$.

TSAIM: the model

- ▶ N particles in a periodic 2d lattice of size $L_x \times L_y$. Average density: $\rho_0 = N/L_x L_y$.
- ▶ Equal population of both species, $N_A = N_B = N/2$.
- ▶ j^{th} particle on site i equipped with a spin $\sigma_i^j = \pm 1$ and a species $s_i^j = \pm 1$.
- ▶ Local Hamiltonian (Ising interaction) on site i :

$$H_i^{(1)} = \frac{1}{2\rho_i} \sum_{k \neq j} J_{jk} \sigma_i^j \sigma_i^k,$$

with $J_{jk} = J_1 s_i^j s_i^k$ (ferromagnetic if same species, anti-ferromagnetic if different species).

- ▶ Flipping rate for the spin-orientation $\sigma \rightarrow -\sigma$:

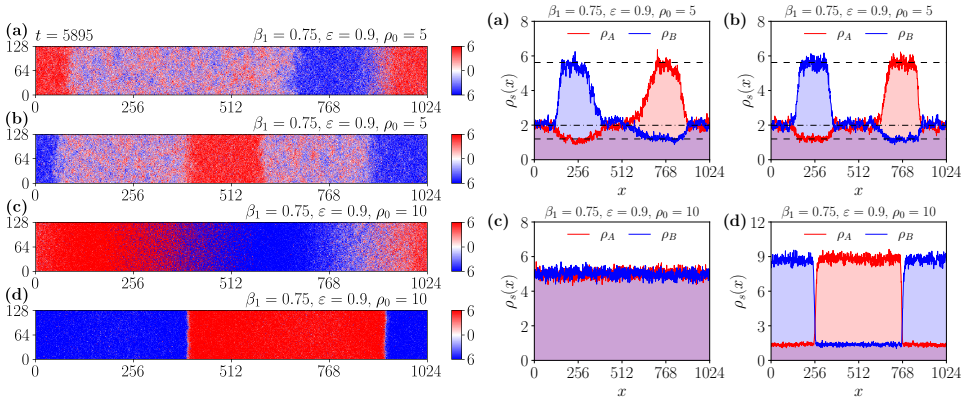
$$W_{\text{flip}}^{(1)} = \gamma_1 \exp \left(-\frac{2\beta_1}{\rho_i} s \sigma v_{a,i} \right),$$

where $v_{a,i} = \langle s_i \sigma_i \rangle = m_{A,i} - m_{B,i}$ is the order parameter.

- ▶ Nearest-neighbor biased hopping in $\pm \mathbf{e}_x$ [const. hopping rate D along $\pm \mathbf{e}_y$]:

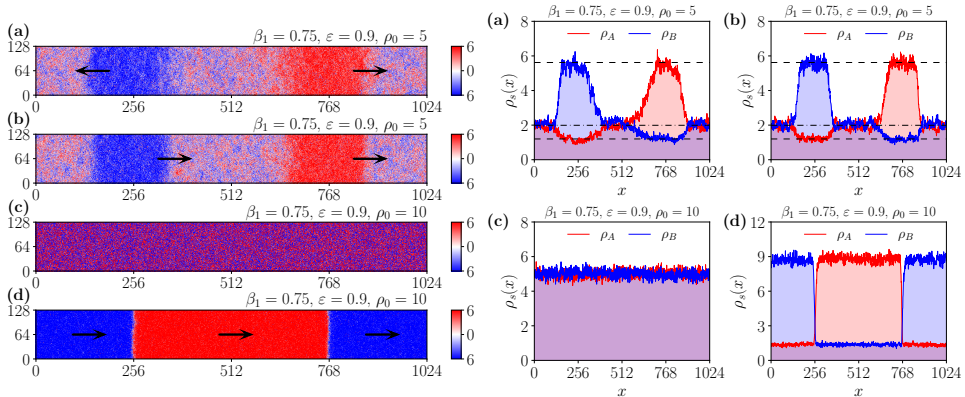
$$W_{\text{hop}}(\sigma) = D(1 \pm \sigma \epsilon).$$

TSAIM: density profiles



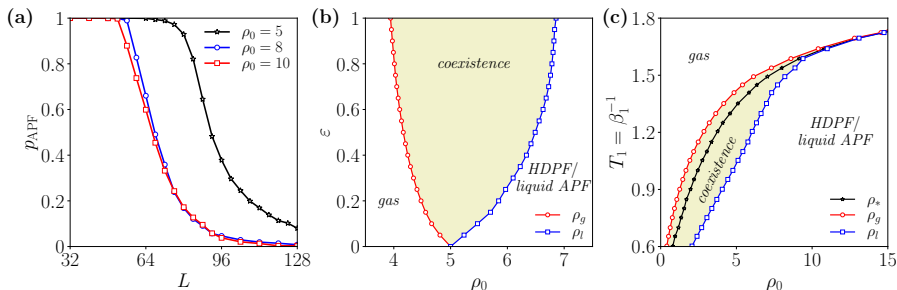
- PF and APF states are present (similar to TSVM).
- The liquid phase can be in both PF/APF states (different from TSVM).
- No stochastic switching between APF and PF states (different from TSVM).
- Explanation: macrophase separated bands, due to normal fluctuations.

TSAIM: density profiles



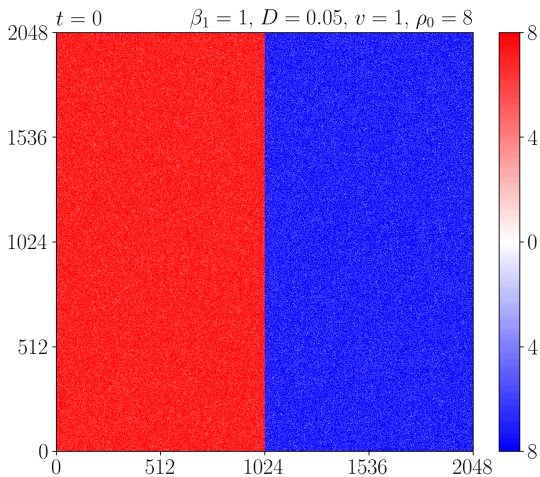
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TSAIM: phase diagrams

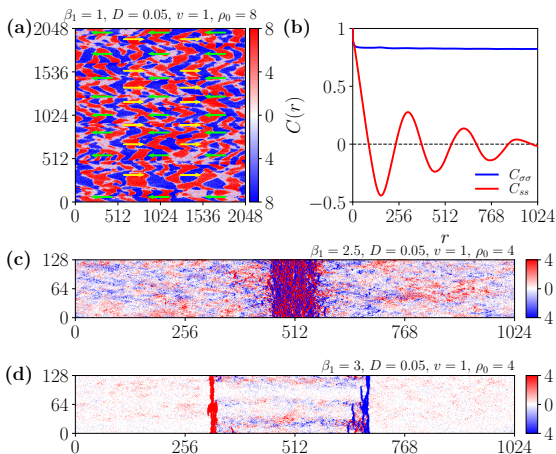


- PF state is mainly obtained during a quench, when $L \ll 1$ (different from TSVM).
- Liquid-gas phase transition, with both PF and APF steady states, depending on the initial condition.

TSAIM: metastability (small diffusion)



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- New steady state with long-range order in spin, but short-range order in species.
- Motility-induced interface pinning when $\beta_1 \gg 1$ and $D \ll 1$.
- Analogous to the one-species AIM. C.-U. Woo and J. D. Noh, PRL **133**, 188301 (2024).

TSAIM with species flip: the model

- ▶ N particles in a periodic 2d lattice of size $L_x \times L_y$. Average density: $\rho_0 = N/L_x L_y$.
- ▶ The population of both species depends on time.
- ▶ j^{th} particle on site i equipped with a spin $\sigma_i^j = \pm 1$ and a species $s_i^j = \pm 1$.
- ▶ Local Hamiltonian (two coupled Ising interactions, active extension of the Ashkin-Teller model) on site i :

$$H_i^{(1)} = \frac{1}{2\rho_i} \sum_{k \neq j} J_{jk} \sigma_i^j \sigma_i^k, \quad \text{and} \quad H_i^{(2)} = \frac{J_2}{2\rho_i} \sum_{k \neq j} s_i^j s_i^k,$$

with $J_{jk} = J_1 s_i^j s_i^k$ (ferromagnetic if same species, anti-ferromagnetic if different species).

- ▶ Flipping rate for the spin-orientation $\sigma \rightarrow -\sigma$ and the species $s \rightarrow -s$:

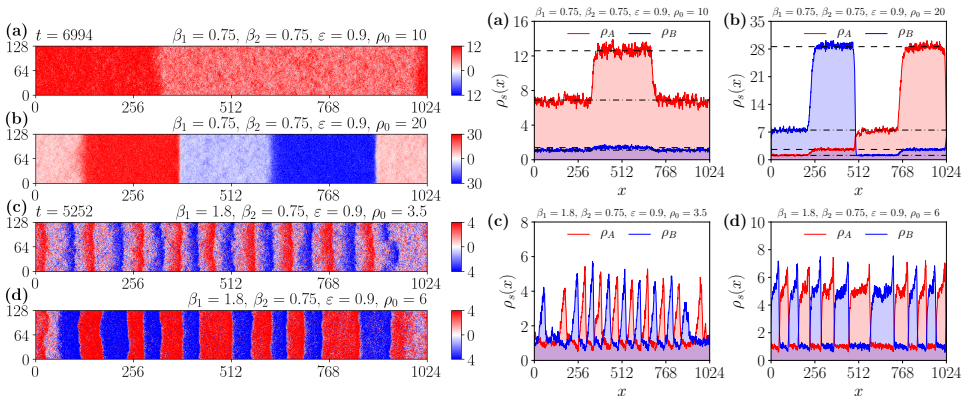
$$W_{\text{flip}}^{(1)} = \gamma_1 \exp\left(-\frac{2\beta_1}{\rho_i} s \sigma v_{a,i}\right), \quad \text{and} \quad W_{\text{flip}}^{(2)} = \gamma_2 \exp\left(-\frac{2\beta_2}{\rho_i} s \mu_i\right),$$

where $v_{a,i} = \langle s_i \sigma_i \rangle = m_{A,i} - m_{B,i}$ and $\mu_i = \langle s_i \rangle = \rho_{A,i} - \rho_{B,i}$ are the order parameters.

- ▶ Nearest-neighbor biased hopping in $\pm \mathbf{e}_x$ [const. hopping rate D along $\pm \mathbf{e}_y$]:

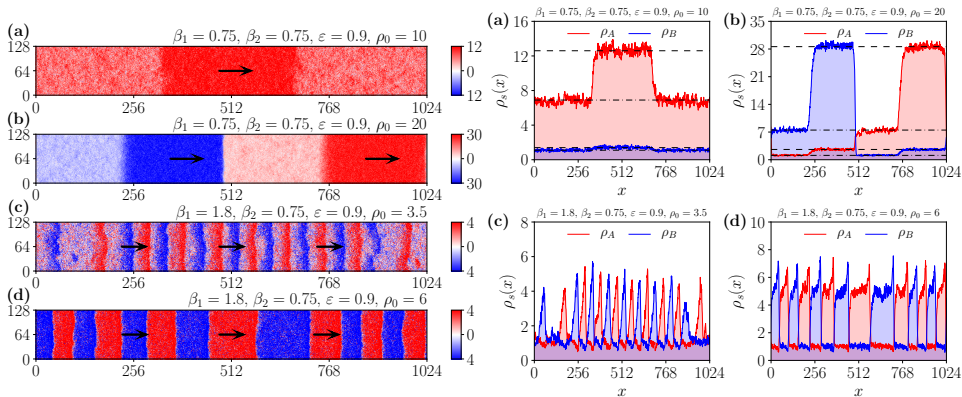
$$W_{\text{hop}}(\sigma) = D(1 \pm \sigma \varepsilon).$$

TSAIM with species flip: density profiles



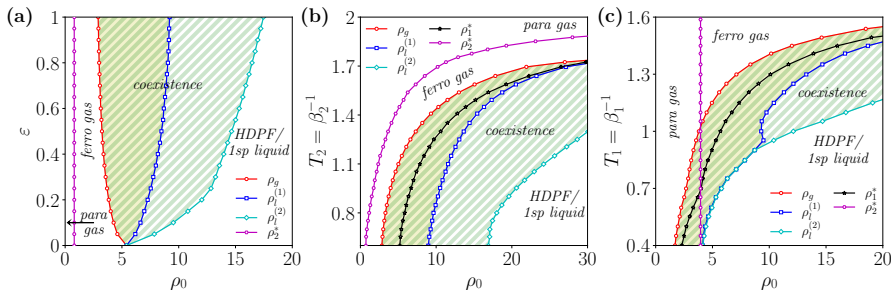
- no APF state.
- Macrophase separated bands when $\beta_1 \lesssim \beta_2$.
- Microphase separated bands when $\beta_1 \gg \beta_2$.
- Liquid phase only in PF state (with one or two species present).

TSAIM with species flip: density profiles



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TSAIM with species flip: phase diagrams

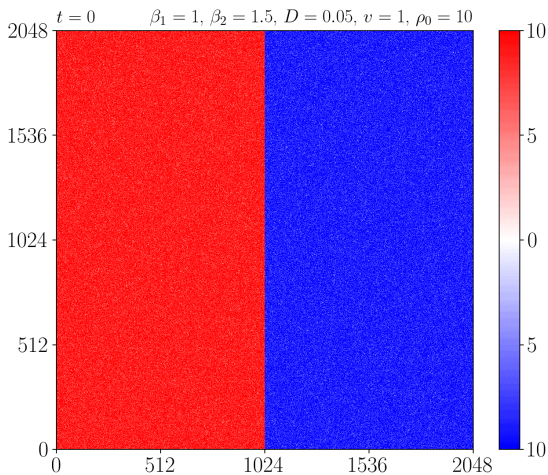


► Phases of the active extension of the Ashkin-Teller model ($\langle v_a \rangle \equiv \langle s\sigma \rangle$ and $\langle \mu \rangle \equiv \langle s \rangle$):

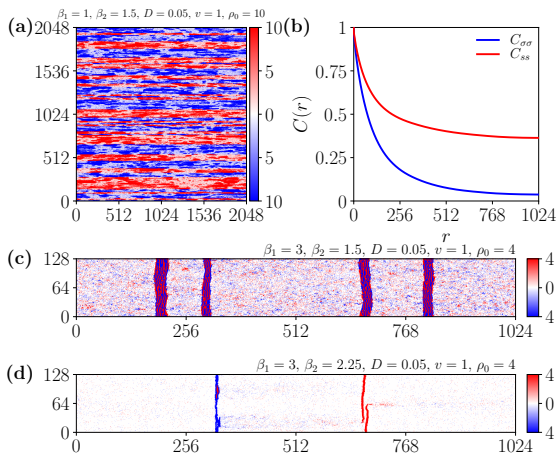
- paramagnetic gas: $\langle v_a \rangle = 0$ and $\langle \mu \rangle = 0$ (spin disorder, species disorder),
- ferromagnetic gas: $\langle v_a \rangle = 0$ and $\langle \mu \rangle > 0$ (spin disorder, species order),
- microscopic flocking: $\langle v_a \rangle > 0$ and $\langle \mu \rangle = 0$ (spin order, species disorder),
- macroscopic flocking: $\langle v_a \rangle > 0$ and $\langle \mu \rangle > 0$ (spin order, species order).

► Flocking is seen as a liquid-gas phase transition, depending on the initial proportion of each species.

TSAIM with species flip: metastability (small diffusion)



TSAIM with species flip: metastability (small diffusion)



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- Motility-induced interface pinning when $\beta_1 \gg 1$ and $D \ll 1$, enhanced with large β_2 .
- Analogous to the one-species AIM. C.-U. Woo and J. D. Noh, PRL **133**, 188301 (2024).

► TSAIM without species flip:

- PF and APF states are present without stochastic switching between both states.
- PF state is formed during a quench in the hydrodynamic limit.
- Liquid phase metastable for small diffusion, forming a new state (LRO in spin, SRO in species).

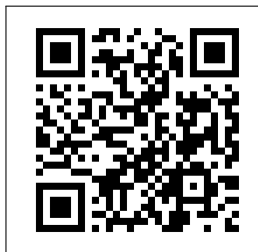
► TSAIM with species flip:

- APF state is absent, but microphase separated bands are formed.
- Active extension of the Ashkin-Teller model (two coupled Ising models), which can be seen as a voter model for self-propelled agents.
- Liquid phase metastable for small diffusion, forming a new state (LRO in species, SRO in spin).

Thank you for your attention !

References:

- ▶ M. Mangeat, S. Chatterjee, J. D. Noh, and H. Rieger, Emergent complex phases in a discrete flocking model with reciprocal and non-reciprocal interactions, arXiv:2412.02501.



- ▶ C.-U. Woo and J. D. Noh, Motility-Induced Pinning in Flocking System with Discrete Symmetry, Phys. Rev. Lett. **133**, 188301 (2024).
- ▶ S. Chatterjee, M. Mangeat, C.-U. Woo, H. Rieger, and J. D. Noh, Flocking of two unfriendly species: The two-species Vicsek model, Phys. Rev. E **107**, 024607 (2023).